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Title: Completion of Contrapunctus XIV: a suitable solution for an inversus quadruple subject-combination.

Keywords: J.S.Bach, Contrapunctus XIV, Die Kunst der Fuge, inverted quadruple subject-combination.

Abstract: Reviewing earlier studies, especially Ferguson, Michael (1990) and Hughes, Indra (2007) into the topic of the inverted quadruple subject-combination in J.S. Bach's unfinished last fugue (Contrapunctus XIV from The Art of Fugue) the author was able to improve their solutions to a considerable extent, and to present a first errorless reconstruction. Herewith the most pivotal problem in the task of completing Contrapunctus XIV seems to be definitely solved. But there are still other...

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Verheijen, Jacques W J M (2016): “Completion of Contrapunctus XIV: a suitable solution for an inversus quadruple subject-combination”.

I: Contrapunctus XIV, the last part and last fugue in Bach’s last masterpiece, “Die Kunst der Fuge”.

The Art of Fugue is an astounding and phenomenal example of Bach’s extreme skills in and control over all elements of contrapuntal technique. At the same time, it is conceptually and intellectually as audacious, daring and sophisticated as was Bach never before; and it is artistically and emotionally deeply lived through: it’s a real masterpiece, his greatest in my eyes. It is Bach at the summit of his Parnassus. It shows us Bach in his full and mature geniality and it is the ultimate and brilliant crown on a lifelong mastership.

There is one central theme, one subject, directly opening the first part of The Art of Fugue, Contrapunctus I, and then governing the whole cycle of 14 Contrapuncti and 4 Canons in all kinds of variations, in an ever increasing degree of contrapuntal complexity, reaching a last and definite climax in the last part: Contrapunctus XIV.

And then there is this sudden and complete anticlimax!

Contrapunctus XIV is a quadruple fugue as we know now. But Bach’s autograph stops at bar 239, in the middle of the third fugue, with an added *Nota Bene* by Carl Philip Emanuel Bach: “Über dieser Fuge, wo der Name B A C H im Contrasubject angebracht worden, ist der Verfasser gestorben.” By now we know that Bach probably deliberately left Contrapunctus XIV unfinished¹.

Directly after Bach’s death much confusion existed about his musical legacy, and especially the status of Contrapunctus XIV. Bach’s publisher didn’t regard Contrapunctus XIV as part, let alone the apotheosis, of The Art of Fugue, and gave it the title “Fuga a tre soggetti”: fugue with three subjects. It took 131 years before Gustav Nottebohm in 1881 discovered that Contrapunctus XIV was meant by Bach as a quadruple fugue, in which the fourth fugue should have the main subject of The Art of Fugue for its subject. And that Bach intended to combine all four subjects into a great quadruple permutation². And Nottebohm found such a combination, in which all four themes fitted together perfectly.

II. The rectus quadruple subject-combination(s).

I present here the four subjects and their rectus combination as discovered by Nottebohm.³ Here are the first three subjects of Contrapunctus XIV as given in Bach’s manuscript:

I



II



III



¹ At least I thought we knew. But Ton Koopman for example, in his comment on a draft version of this article, seriously questioned this hypothesis.

² Nottebohm, G. (1881) “J.S. Bach’s letzte Fuge”, Music-Welt, p. 234

³ Most of the examples in this article are borrowed from Indra Hughes (2006): “Accident or design? New theories on the unfinished Contrapunctus 14 in Bach’s The Art of Fugue, BWV 1080”, with kind permission of the author. Hughes thesis can be found at:

<http://www.indrahughes.com/pdfs/INDRAHUGHESDOCTORALTHESISAccidentorDesignNewtheoriesontheunfinishedContrapunctus14inJSBachsT.pdf>

The fourth subject, main theme of Die Kunst der Fuge:



Nottebohm's combination:

So here Nottebohm found a beautiful positioning and layering of the 4 subjects: they fit together in a perfect way which impossibly can be coincidental. Doubtless: Bach designed them for the purpose of their combination.

Now Nottebohm's rectus quadruple subject combination is not the only one possible. And it could be and is criticized for the consecutive fifths between soprano and alto in bar 3, as for example by Sir Donald Tovey.⁴

Tovey came up with three other rectus permutations. It would go too far in the context of this article to review these combinations. In fact Indra Hughes⁵ has investigated them in his book very scrutinously, so I refer the interested reader to his study at this point. He improved Tovey's work to a great extent.

Hughes himself found two rectus combinations, "that "work " according to the strict parameters that I have set for them (which I believe to be in accordance with Bach's own strict parameters)"⁶

Parameters which he earlier described on page 139 as follows:

"A combination which avoids consecutives, preserves Bach's fugue subjects in their unadulterated form of IV, uses the 14-note form of IV, has a strong bass line, avoids congestion of the voices, is playable by two hands on one keyboard, and is musically convincing."

I would like to share Hughes' second combination here with the reader because of its particularly sonorous quality and beauty⁷:

Ex. 4.30: II-III-IV-I using the 14-note form of IV

⁴ Tovey, D. F. (1931): A companion to "The Art of Fugue" (Die Kunst der Fuge) of J. S. Bach", Oxford University Press, London.

⁵ Hughes, I.: op. cit., especially Chapter IV

⁶ Hughes, I.: op. cit. page 141 - 142

⁷ Hughes, I.: op.cit. page 141.

The ultimate answer on the question about how many rectus permutations, and in which layerings and keys Bach would have incorporated in the fourth fugue of Contrapunctus XIV, I leave here undiscussed as beyond the scope of this article: it is not my focus here.

III. The search for an inverted quadruple subject-combination.

The Obituary, the “Nekrolog”, written shortly after Bach’s death by C. Ph. E. Bach and Johann Friedrich Agricola, and published by Mizler in 1754, makes the following statement about The Art of Fugue and Contrapunctus XIV:

*“Dies ist das letzte Werk des Verfassers, welches alle Arten der Contrapunkte und Canonen, über einen einzigen Hauptsatz enthält. Seine letzte Krankheit hat ihn verhindert, seinem Entwurfe nach, die vorletzte Fuge völlig zu Ende zu bringen, und die letzte, welche 4 Themata enthalten, und nachgebends in allen 4 Stimmen Note für Note umgekehret werden sollte, auszuarbeiten. Dieses Werk ist erst nach des seeligen Verfassers Tode ans Licht getreten”.*⁸

On the basis of this statement, we can be pretty sure that Bach’s master plan for the completion of Contrapunctus XIV included not only the construction of one or more rectus quadruple subject-combinations, but also the construction of at least one inversus combination of the four subjects: a real master plan indeed.

A lot of Bach-scholars, fascinated by this problem of the unfinished Contrapunctus, have tried to find a suitable completion. And there appeared much sensible and nonsensical material on this subject in those 266 years after Bach’s death.

I’m the next one preparing a book on the subject of the completion of Contrapunctus XIV.

There are a lot of problems to be solved yet, because there isn’t much agreement between Bach-scholars about all the issues at stake here.

But within the framework of this short article I focus on only one, but fundamental and pivotal, problem in the solution of this ultimate enigma: finding a suitable inverted quadruple combination.

Finding possible and correct quadruple subject-combinations in rectus-form was rather peanuts compared to the task of finding a suitable inverted quadruple combination. Most completers of Contrapunctus XIV didn’t even try to find one, or were satisfied with less strict and far-reaching attempts (for example a combination of IV inverted with I, II and III rectus). Others who tried were very disappointed by their results, because their solutions didn’t sound like Bach at all!

Indra Hughes pays much attention to this problem and gives a good overview in his book of the more serious attempts to find an inversus quadruple combination. He comes up with an extensive analysis of the strong and weak points in the two inverted quadruple combinations found by Sir Donald Tovey in the thirties and one developed by Michael Ferguson in the nineties. And on the basis of this analysis, he proposes a series of important and valuable improvements, which eventually results in his own inverted quadruple combination.

It was in fact the American organist Michael Ferguson, who found the key for the solution of the inversion problem by thinking “out of the box”, by taking a quote from Carl Philip Emanuel Bach in the Obituary very literally. I repeat the quote:

⁸ English translation: “This is the last work of the author, which contains all sorts of counterpoints and canons, on a single principle subject. His last illness prevented him from completing his project of bringing the next-to-last fugue to completion, and working out the last one, which was to contain four themes and to have been afterward inverted note for note in all four voices. This work saw the light of day only after the death of the late author”. David, H.T., Mendel, A., et al.(1998): The new Bach reader: a life of Johann Sebastian Bach in letters and documents, New York, W.W. Norton, page 304

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Now Ferguson showed the way towards the solution by interpreting the words “...and to have been afterward inverted note for note in all four voices” very strictly, namely as semitone for semitone. All his predecessors who had tried to invert the four subjects into a combination, had done so on the assumption that the mirroring had to take place in the same D minor scale as the rectus combination. Take for example subject IV rectus and inversus:

Rectus

```

      A
    F
  D   D
      D
      E
      F
      G
      F
      E
      D
  
```

Inversus

```

      Bb
    A   A   A   A
      F       G   G
      D       F   F
              E
  
```

Not so strange that they tried this: Bach himself used this inversion of subject IV several times in The Art of Fugue, for example in Contrapunctus II, V and XII. But where this works perfectly with subject IV, it does not work very well in the combination of the four subjects together: the result is invariably rather unconvincing and mediocre, and certainly not meeting the qualitative standard Bach had set for himself in writing The Art of Fugue and his last fugue.

Michael Ferguson’s argument was that this isn’t an exact mirroring. And it is rather easy to see why. Because where the first interval is an exact mirror (a perfect fifth), the second is not: A – F in the rectus form is a major third, but D – F in the inverted form is a minor third. So Ferguson corrected the first two bars as follows:

Rectus	Inversus
A	A A
F	F#
D D	D

What strikes immediately is that subject IV opens here suddenly and surprisingly with a major triad instead of all the former minor settings. The other subjects I and II also open now in a major setting, subject III remaining neutral in this respect because it does not contain the third of the scale. It strengthened Ferguson that he was on the right track, as it did Hughes (and myself). But Ferguson’s result wasn’t satisfactory enough: it sounded harmonically too illogical, confused, unbalanced and un-Bachian.

And although Hughes' improvements were considerable, and led to a better end result, the same still applied to his solution.

IV. The problem with Ferguson's and Hughes' inversion.

And then I found out where things had gone wrong. It turned out that Ferguson hadn't been as strict and consequent in his semitone for semitone mirroring as he had intended. He had violated his own principle of a strict semitone-for-semitone mirroring.

(We have to take into account that Ferguson presented his inverted combination not in D, but in Bes for good reasons. So this asks for a transposition, but it doesn't change the problem.)

Ex. 4.53: Michael Ferguson's inverted quadruple combination

Source: Indra Hughes, *op cit.* Page 161

Now this opens absolutely surprising, convincing and good: the inversion not only opens with a major triad but establishes a new major tonality with a clear cadence in the first bars, ending in a deceptive vi: I(maj) – IV (maj) – V7 – ii.

I can imagine that this gave Ferguson (and Hughes) an ultimate "Eureka"-sensation.

But it is a self-organized trap. For where Ferguson writes "G" as the fifth note of subject IV and as the last note in bar 1 of subject II, a correct semitone for semitone mirroring prescribes "G flat", resulting in a slightly but fundamentally different progression: I(maj) – IV (min) - V(7) – vi.

In fact Hughes copied Ferguson's error into his own solution. Let us follow Hughes step by step.

On page 160 he gives us in Ex. 4.52 a correct inversion of all four themes semitone for semitone:

Ex. 4.52: The four subjects inverted semitone for semitone

Then in Ex. 4.53 he gives us Ferguson's inverted quadruple combination, as was shown already above.

And after a meticulous series of improvements, Hughes comes up with his own inversion, in which Ferguson's "G" is still prominently present:



Ex. 4.58: The combination complete with suggested non-thematic material

Note that Hughes does not use the initial 12-note version of subject IV from The Art of Fugue, but the 14-note version, which Bach also used frequently later on in The Art of fugue. I fully agree with Hughes that the 14-note version of subject IV is a much better choice for the inverted combination than is the 12-note version.

In an earlier draft of this article, which I published on my website, but will be removed and replaced by this definitive version, I wrote that Hughes unconsciously copied Ferguson’s mistake, and overlooked that “G” on the first beat of bar 3 had to be “G flat”.

But this is not the case. In fact Hughes writes on page 161 of his thesis the following lines about Ferguson’s inversion:

*“Ex. 4.52 sets out the subjects inverted semitone for semitone as Ferguson suggests, **rendering each one in the major mode** (my bold, J.V.)”⁹*

*“There can be no doubt that this sounds very much more convincing than any of the inverted combinations we have seen hitherto. There are a couple of details to mention about subject II: the last (tenor) note of bar 1 is a G[♯] where a strict semitonal inversion from the original C[♯] (before transposition into B[♭] major) would have resulted in a B[♭] (i.e. G[♭] after transposition). **This minimal alteration is necessary to avoid introducing any sense of the minor mode that a G[♭] would bring into a B[♭] major tonality** (my bold, J.V.).”¹⁰*

And this is after he has quoted Ferguson with assent:

“An inversion of all four themes seems required by statements made by those who had knowledge of Bach’s intentions. And yet, if inverted as is typically done (in the minor mode), this has an unlvely effect that no-one has ever believed Bach would approve for use in his greatest fugue. However, if strictly inverted - semitone for semitone, something no-one before me has attempted – the mode changes tot he major, and acquires that darkly brooding quality that only Bach could achieve while writing in a major key.....”¹¹

Now we arrive at the heart of the problem, and I can show why both Ferguson and Hughes did not succeed in finding the right solution for it. Why both did not succeed in grasping Bach’s genial logic when he constructed his quadruple inversion.

Both authors are victim of their own incorrect and unsubstantiated assumption: if it is not the minor mode it has to be the major mode!

As a consequence of this wrong assumption they have to violate their own key for the solution: a strict semitone for semitone mirroring. And so they can *“avoid introducing any sense of the minor mode that a G[♭] would bring into a B[♭] major tonality”* for the first 4 bars. Only to discover that in the end, in bar 5 and 6 this G[♭] cannot be avoided at all. It did not result in rethinking their assumption.

⁹ Hughes: op. cit. page 160.

¹⁰ Hughes: op. cit. page 161.

¹¹ Ferguson: “Letter to the editor”, The American Organist 35(4), page 18 – 24.

V. Bach's genius again.

And this is the main cause of the structural tonal unbalance of their solutions. For what they tried to avoid (and cannot in the end) was exactly what Bach was looking for and found, and what gave him a perfect solution. Bach did not write his inverted combination in the minor mode, nor in the major mode: he combined them! The scale Bach used was neither minor nor major, but Moll-Dur, the harmonic major scale, the one with the major third and a minor sixth.

Now at one stroke all inconsistencies disappear, all tonal confusion has gone, and the inversion develops itself in a rich and beautiful harmonic progression and in a recognizable Bachian logic and language. There is no necessity to violate the key for the mirroring process. On the contrary: we can and have to apply it as strictly as possible in order to find a solution for Bach's enigma.

So here, in the opening of the inverted combination, Bach has surprised us and led by the nose two times for 266 years: one directly after the other. First by his apparent escape from the minor mode, and second by escaping from a formal minor-major dichotomy, and coming up with something much more interesting.

When we substitute IVmin for IVmaj in the opening sequence I – IV – V7 – vi we in fact restore, or better: reveal the inherent harmonic logic which has to be the architectural base of the whole passage. It was, remember, the IVmaj which initially seemed so promising, but turns out to be the source of a structural harmonic confusion in the passage under consideration.

We now not only have a perfect mirroring of the four themes in a vertical inversion: as a kind of Bach-bonus the substitution also creates a horizontal reversion, since the passage opens with I – IVmin and closes reversed (again by necessity) with a plagal cadence: IVmin – I.

I think we are finally on the right track: nearing Bach's idiom and his profound architectural way of designing musical content and form in perfect balance.

There is something very amusing in all this. I see Bach, developing his bold inverted combination, chuckling in the knowledge that his completers would have to struggle very hard and make this kind of mistakes, based on their own (understandable and obvious, but incorrect) assumptions.

And this also confirms that I'm on the right track: not only because of the necessary corrections, but also because it is very characteristic of Bach's writing that he continuously surprises and amazes us by doing the unexpected: as if he is purposefully playing with our assumptions and expectations.

It maybe does not lend further support to the hypothesis that Bach purposively left Contrapunctus XIV unfinished in order to create a phantastic musical enigma. But if he did, we now can reconstruct how he did this, in what ingenious way he designed the greatest musical enigma ever in Western musical history. At least as far as the inversion problem is concerned.

VI. How easy to fall into the trap.

I have to admit that it took considerable time and effort before I discovered Ferguson's and Hughes' error. The first bars of their inversions seemed so plausible and sounded so good, that I too completely overlooked this error. I played Hughes' combination over and over again, knowing and feeling that something was wrong here. But since his and Ferguson's combination opened with that convincingly bright and festive version in major, I thought the problems with these combinations were in the rear, especially in bar 6, where the passage comes to an unsatisfactory end, leaving the listener behind in confusion about the tonal structure of the whole.

Now there **are** some real problems in the rear, mainly caused by the free notes which Hughes added to the soprano there, and which contribute considerably to the overall and already present confusion and harmonic imbalance, which I shall investigate and discuss in the next paragraph.

But when I skipped Hughes' free added notes in the soprano, the sensation of harmonic imbalance didn't disappear. I subsequently improved the last bars by finding better free notes for the soprano. And first when even these improvements didn't have a sufficiently satisfactory and convincing result in overcoming the problems of structural imbalance, I began to realize that the main problem was not in the rear but in the bright and shining start of the combination.

So it is in the end this tiny but significant change on the first beat of the soprano's third bar (and the last note in the tenor's first bar), where IVmaj has to be substituted by IVmin, which with astonishing simplicity restores the architectural and harmonic balance of the whole passage, in a way which is perfectly consistent with Bach's own criteria for the inverted combination, both conceptually and artistically. It's also consistent with scientific deduction, and with Ferguson's and Hughes' insight on the semitone-for-semitone mirroring: they were on the right track, which they unfortunately didn't follow consequently. Nevertheless they have to be fully credited for their important contributions. I only pursue their quest, standing on their shoulders, and operating as a corrector of their inconsistencies.

In fact, discussing my point of view with Indra Hughes by e-mail, he was very supportive. He also admitted frankly that G should be G flat. Of course, apart from this, his combination of the subjects as such keeps its validity.

In my opinion this was the last step we had to take yet, in a long process of finding a suitable inversion. With this last step the most pivotal problem in how to complete Contrapunctus XIV has basically been solved. There are still a lot of other problems to be solved, which are not discussed here within the framework of this short article. I hope to solve some of them in my forthcoming book "Would Bach have done this?"

I'm not claiming that "this is **the** answer". I'm not claiming that Bach would have done this. But at least he could have done this. Maybe he has yet some other surprises in store for us. Maybe my solution has to be criticized and improved on details. Because of the free notes in play there are other and maybe better solutions possible.

But my solution offers an important improvement compared to all former solutions for an inverted quadruple subject-combination, as far as I know them. And, most important of all: it finally sounds like Bach again.

So I came to the following solution for the inversion of the four subjects:

The image displays a musical score for the inversion of four subjects. It consists of two systems of staves. The first system shows the beginning of the piece, with a treble clef staff and a bass clef staff. The second system starts at measure 4, indicated by a '4' above the treble clef staff. The music is written in a key with one flat (B-flat major or D minor) and a 3/4 time signature. The score features complex counterpoint with multiple voices and various rhythmic patterns.

VII. Problems in the rear of the inversion.

I mentioned earlier the problems in the sixth bar of Hughes' inversion. They are mainly the consequences of the free notes he has added in the soprano, after subject IV has come to its end on the first beat of bar 5, and thus do not pertain to the essential thematic material of the subjects.

Hughes writes “D flat” in the soprano on the last beat of bar 5 and on the first beat of bar 6. This results in an unpleasant, uneasy and unmediated dissonance and oscillation with the unavoidable “D” (as part of subject II) in the second beat of bar 5 in the tenor, and with the unavoidable “D” (also as part of subject II) in the second beat of bar 6. Since there is no good reason for writing “D flat” in the soprano here, we can better make another choice.

Something similar holds for A natural or flat (both indicated) on the last beat of bar 6: in uneasy dissonance with the unavoidable A flat (as part of subject III) in the alto, and also with the A flat on the first beat of bar 7: at least I can’t see any reason why Bach would opt here for a dominant seventh chord on the tonic. And it results in an unpleasant and unstable tonal oscillation, which is harmonically dysfunctional. All these vicissitudes can be easily avoided, and by doing so the whole passage regains its necessary tonal stability and clarity.

But I still wasn’t wholly satisfied with my solution. It still didn’t sound completely logical and natural. I discussed it with Indra Hughes. He asked me “How do you analyze G flat in the alto in bar 6?” He meant the first one of the two in this voice.

Now for me, the function of this note is perfectly clear: it’s a suspension (originating from the preceding chord range on the first and second beat and interpretable as part of F9 in inversion and/or part of C7 (ii in B flat): and resolving upwards into A flat as a component (seventh) of a secondary dominant on B flat (V7) resolving on its turn into E flat minor(IV), which eventually leads to a plagal cadence back to B flat major.

VIII. Making the same mistake myself.

And then I realized that I had made a similar kind of mistake as Ferguson and Hughes did.

For if my analysis is correct (and I don’t have any doubt about that), then D natural in the tenor in the second beat of bar 6 is perfectly logical and functional, being part of V7 of the following E flat minor.

But this is not the case with the D natural in the soprano in the third beat of bar 6. Bach, using a minor subdominant, certainly would opt for D flat here, as logical and functional, and as being congruent with the minor character of this subdominant.

Stated in another way: it’s one of the characteristics of Bach’s style that he can freely and flexibly alternate between or switch from major to minor and vice versa. See for example how he did this earlier time and again in the bars 46 up to 69 of Contrapunctus XIV. But these alternations are always logical and functional in a broader harmonic context.

Now where I criticized Hughes’ solution for this reason, I also had to criticize my own solution for the very same reason: lack of insight in functional harmonic clarity. It’s here also all about just one semitone. The same tone, the one moment perfectly functional, the other moment already wrong because the logic of the harmonic development has changed in the meantime, and asks for another choice.

Of course, I could avoid using D natural or D flat in the soprano by substituting for example an F for it. But this leads also to unsatisfactory results in the further progression of the soprano part. I’m pretty sure, that Bach hasn’t tried to avoid this: on the contrary, he has probably used the D flat here deliberately in order to re-establish, to reinforce and to deepen the richness of his tonal logic.

A last remark: the voice-leading in the soprano ends in a final D natural on the first beat of bar 7. This results in a doubling of the third in the B flat chord. Although this doubling is not forbidden, we had rather opted for a doubling of the tonic or the fifth here. But all trials into that direction are blocked, since they all result in (hidden) consecutive fifths or octaves. So doubling the third here is the only possibility.

It would have been less acceptable if this had been the final chord of the piece. But here, where it is only a moment in transition, on its way into a new development, it is no problem at all.

So I can conclude this short article now with a final version, to be presented as a suitable solution for the problem of the construction of an inverted quadruple subject-combination in Contrapunctus XIV:

Final solution for an inverted quadruple subject-combination

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